

NAME: \_\_\_\_\_

1. Prove that:

$$\sum_{i=1}^n (2i + 3) = n^2 + 4n$$

2. Combine the following permutations. Remember  $f \circ g(x) = f(g(x))$ .

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 1 & 7 & 6 & 5 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 2 & 1 & 3 & 7 & 6 \end{pmatrix}$$

3. Convert the following numbers from binary to hexadecimal.

10111011001010

1111000010000001

1111011011

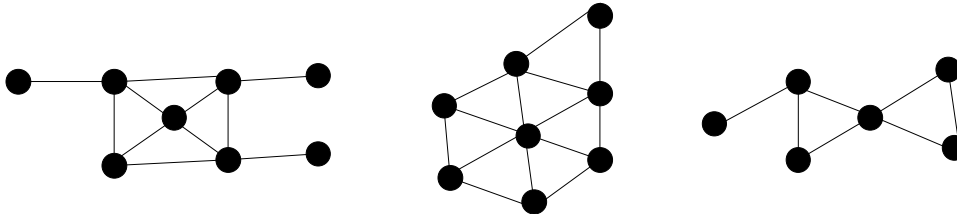
111010101100100

4. Give a minimal Boolean expression for the following function, F.

|     |   |
|-----|---|
| TTT | F |
| TTF | F |
| TFT | F |
| TFF | T |
| FTT | T |
| FTF | T |
| FFT | T |
| FFF | F |

5. I roll three dice. What are the probabilities of the following events?
- I get three ones.
  - I get exactly two sixes.
  - I get either exactly one six or exactly one five (or both).
  - I get no sixes.

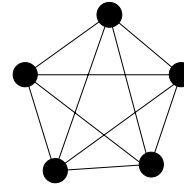
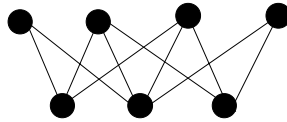
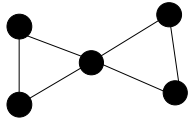
6. Do the following graphs have Hamiltonian paths, Hamiltonian cycles, or neither?



7. In the list of all permutations of the digits 1,2,3,4,5,6 what follows 3,6,5,1,4,2? In the list of all combinations of three digits from the set  $\{1,2,3,4,5,6\}$ , what follows 2,5,6?

8. In the polynomial  $(x+1)^{22}$ , what is the coefficient of  $x^{19}$ ?

9. What is the chromatic number of the following graphs?



10. Decrypt the following message, and have a Blessed Christmas. (You do not have to finish this here. You may remove this page and take it home. (Automatic 10 points for this question.)

GUFOFHTOF GUF PTOJ URYBFPH BUKPP ARLF DTN K  
BRAV; WFUTPJ, K LROARV BUKPP XTVXFRLF, KVJ  
WFKO K BTV, KVJ BUKPP XKPP URB VKYF RYYKVNFP  
-- ATJ IRGU NB.

RBKRKU 7:14